Toobeez Math Activity Workbook

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Award-Winning!

Tested in 2004 by multiple teachers with families, the Toobeez Interactive Family Edition has won a Teacher's Choice Award for 2005. The Interactive Family Edition is what you get when you combine a set of Toobeez with Tom Heck's book, *The Official Toobeez Team Building Games and Activity Guide*, and Multimedia Training CD.

All Project Connect Joint Venture participants are members of the following organizations

Association for Experiential Education

NSSEA
NATIONAL SCHOOL SUPPLY AND EQUIPMENT ASSOCIATION

american CAMP association
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Project Connect
Training Opportunities

Do you want to:

- Become a more effective educator, teacher or trainer?
- Learn to creatively increase student involvement?
- Develop strategies for creating a dynamic learning environment?

If you answered yes to any of the above questions, then the Project Connect training workshops are for you! We offer two types of workshops:

1. **Train-the-Trainer Workshop**
2. **TEAM Workshop**

Train-the-Trainer Workshop

This workshop is for you if:

- You are an educator looking for new ways of teaching team and leadership skills
- You like learning in a fast-paced and hands-on environment

What to expect

In the Train-the-Trainer workshop, you will learn how to lead activities that develop team and leadership skills. This workshop is fun, empowering and educational. It is also experiential, meaning you will learn by doing.

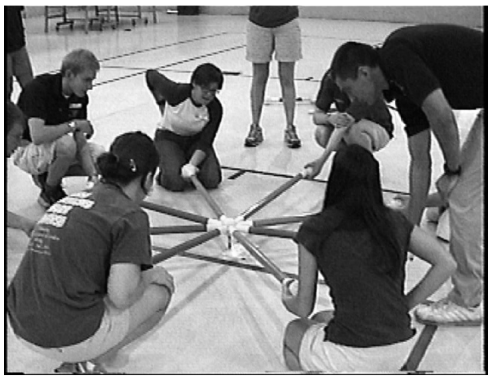
You will learn activities that promote trust and creative problem solving, as well as encourage purposeful and kind communication. You will experience physical activities, as well as activities which demand high cerebral skills. You will learn how to

design everything from a five-minute icebreaker to a full two-day adventure. Depending on the length of the program you choose, you can even learn how to integrate the activities into an existing curriculum.

You will learn how to deliver important processing skills to learn how to draw out the genius of the group by asking thought-provoking questions. When a discussion session is delivered properly, groups learn to own the information they generate from the teambuilding activities, providing them with a sense of empowerment, connectedness and success!

This is the perfect workshop for you if you are a:

- Classroom teacher
- School counselor
- Camp counselor
- Human Resources director
- Trainer or workshop presenter
- Therapeutic Recreation Specialist



TEAM Workshop

TEAM = Together **E**veryone **A**chieves **M**ore

The TEAM workshop is for your team if you are ready to:

- Move quickly from a "good" team to a "great" team
- Have fun and be challenged in a dynamic learning environment
- Be treated with respect and dignity while leveraging the strengths of your group

Over the years, we have worked with thousands of people from all walks of life, including:

- Business teams (from such industries as investing, communications, and insurance)
- Non-profit organizations (such as YMCA, YWCA and Boys & Girls Clubs)
- Educators (including public & private school teachers, school counselors, college students, college teaching staff and international students)
- Alternative education programs (including corrections officers, after-school programs and enrichment programs)

What to expect

In this workshop, you will participate in fun activities and engaging hands-on learning exercises to help your team address vital issues so it can move forward. This workshop focuses on where you are now as a group and where you are going.

For more information about attending or scheduling teambuilding training workshops, call:

1-877-TOOBEEZ

1-877-866-2339

Toobeez®
PROJECT CONNECT
Fostering Connections and Teamwork Through Fun!

Table of Contents

Introduction

Project Connect is dedicated to providing cutting-edge activities and tools for educators. Toobeez are a unique means of teaching academics, social skills, teamwork and problem solving. For the first time, the Toobeez program has been adapted as a useful teaching tool for utilization in schools and educational forums. The activities found in this workbook are designed to challenge students to evaluate concepts from a practical point of view with hands-on learning opportunities.

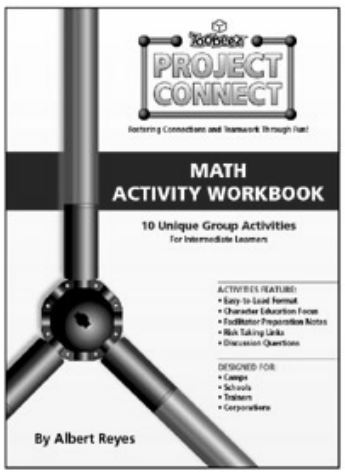
Project Connect Materials

Project Connect offers the following products:

Toobeez

Toobeez are the incredible construction tubes used to build and create the activities for your group! A one-of-a-kind construction system, Toobeez give you the chance to "connect" as a group while engaging everybody in constructive play. Toobeez are a hands-on, easy-to-assemble and safe product that captivates the imagination of anyone who uses them. You can use them to hold a powerful learning experience anytime!

Each 57 Piece Toobeez Kit contains: (20) 3.4" spheres, (8) 36" tubes, (8) 24" tubes, (8) 16" tubes, (8) 11" tubes, (1) GIANT Toobeez storage bag, (2) red slide-on curtain panels, & (2) blue slide-on curtain panels. Shipping box is 36" x 12" x 6" and weighs 21lbs.



Activity Workbooks

The *Toobeez Math Activity Workbook* includes 11 detailed lesson plans with safety reminders, helpful hints, procedures, discussion questions and more. Other available products include Language Arts, Physical Education, Science, Occupational Therapy and Teambuilding workbooks. A teambuilding Multimedia Training CD is also available.

The Teacher's Role

The teacher plays a crucial role in the Toobeez program's activities. After selecting the activity, the teacher must aid students in executing the instructions, monitor safety, and observe the group's dynamics. Most important, it is the teacher's responsibility to convey positive leadership and the potential for success to create the best environment for these academic activities.

Motivational Learning Guide
Once the teacher determines that an activity is suitable for a particular group, he or she must make judgments about how the group is working. The teacher is responsible for running the activity; however he or she should not run the group. Instead, the teacher should act as a guide for students and ask probing questions which will help the group to progress in the activity. The teacher should positively motivate students so each individual enjoys the best possible learning experience.

Observer
While running the activity, the teacher must allow students to solve problems on their own. When groups are working, the teacher should take a step away and observe the group's dynamics so he or she can guide the group appropriately if they encounter problems.

Discussion Coordinator
The teacher begins the discussion by asking the questions he or she feels are most appropriate for the group. While considering the lessons learned from the challenge, as well as the possible "failures," the teacher should focus on the group's positive learning experiences while acknowledging both negative and positive issues.

Connections to Education

Toobeez are a unique means of teaching social skills, teamwork and problem solving. For the first time, the Toobeez program has been adapted as a useful teaching tool for utilization in schools and educational forums. The lesson plans developed with Toobeez are designed to challenge students to evaluate concepts from a practical point of view with hands-on learning opportunities.

Teachers can use Toobeez as an innovative tool to implement the current educational practices discussed below. In addition, use of Toobeez in the classroom can play an important role in establishing connections between students, peers and teachers!

Brain-based Research

The Toobeez program activities support brain-based research by engaging learners and providing enjoyable hands-on experiences. The essence of brain research suggests that all learners have emotions, desires and needs. Researchers have found that relaxed, yet active, lessons that completely immerse the participants provide the most authentic learning experiences. This type of environment has been shown to increase retention, enjoyment and positive feelings about learning. The Toobeez program provides activities that challenge each participant at his or her individual level while engrossing them in a fun-filled learning environment.

Multiple Intelligence Theory

Developed in 1983 by Dr. Howard Gardner, the Multiple Intelligence Theory states there are multiple learning styles that maximize the learning potential in children and adults. The theory takes into account the various strengths in individuals, such as linguistic, logical, spatial and kinesthetic skills. It also suggests that different people learn in different ways. The Toobeez program provides a unique way to broaden the learning experience beyond traditional methods. By allowing teachers to offer lessons which address different learning styles, learning increases across the classroom.

Differentiated Instruction

In addition to building on the varying strengths of individuals, the Toobeez program allows instructors to appropriately adapt lessons to the various ability levels in a classroom. Activities can be enhanced for a greater challenge or they can be modified for a simpler activity.

Problem Solving

Lesson plans and activities in the Toobeez program require critical thinking and problem-solving skills. Participants are required to work together to brainstorm, select and execute solutions to each activity challenge.

Character Education Themes

Helping young people develop good character is a goal of many educational settings nationwide. The Toobeez program offers activities that can supplement or launch a character education curriculum. The character themes listed below have been aligned with the activities in this guide, and the first page of each activity associates a character education trait with the activity.

- Perseverance	- Teamwork/Cooperation	- Uniqueness
- Respecting Others	- Citizenship	- Caring
- Communication	- Responsibility	- Trust

Teambuilding & Connections with Others

Research has shown that a sense of "connectedness" to parents and peers is the most influential protective factor in a teenager's life. If teenagers lack this connection, their chance of engaging in risky behaviors increases. Through increasing teambuilding skills, the Toobeez program brings peers together while boosting the individual's self-confidence and sense of "connectedness" with others.

Collaborative Hands-On Experience

The Toobeez program centers around a collaborative learning environment. Groups work together to solve each challenge while completing interactive, hands-on activities. Each lesson involves member participation, movement and teamwork for completion of the activity.

Safety

Please use caution when utilizing Toobeez in your classroom. Studying the material in this activity guide is not a substitute for professional training. Please refer to page 6 for information regarding the training options offered by Project Connect. For additional safety information, as well as product assembly and care, please turn to page 86.

Safety Tips!

- Do not allow students to climb on any Toobeez structures
- A first-aid kit should be easily accessible during the activities
- Follow general classroom safety guidelines during all lessons

The information presented in this activity guide is a reference, and the teacher is ultimately responsible for judging the suitability of an activity and safely supervising the activity.

The teacher's job is to make safety a priority and to manage the risk.

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How to Use This Workbook

Lesson Introduction Page
The first page of each lesson contains an outline of information for the teacher. This page includes lesson objectives, the activity challenge and links to character education. An **Activity Plan** box on the page includes overall information about the activity. *Please note: The time listed in this box is the total estimated time for the lesson (including time to setup or conduct group discussions prior to the activity.) Space requirements are defined as follows:

Minimal: The activity can be conducted in the front of the room
Medium: A few desks may need to be moved
Lots: Clear all desks or relocate to the gym or an outdoor area

Activity Setup
This section provides step-by-step instructions for the teacher or students to follow for Toobeez construction. Some setups also include mini-lessons to review or introduce concepts that are utilized in the activity.

Helpful Hints
This section provides the teacher with suggestions to help guide the students. Also, tips are offered to avoid possible obstacles.

Activity Instructions
This section explains how to conduct the actual lesson.

Activity Challenge
Within each activity's instructions, a challenge is presented to the students. Be sure to read this box to the group and clarify their understanding of the challenge.

Activity Discussion and Processing
This section provides the teacher with a discussion topic that relates to the activity, as well as questions to use in guiding a group discussion.

Activity Variations
This section provides variations of easier or more challenging versions of the activity for the group's use.

Notes
Space is provided for the teacher's notes on the activity. Notes can help teachers reflect on the lesson, as well as record possible future modifications.

Learning Strategies

A number of learning strategies can be used in conjunction with Toobeez classroom activities. Prior to an activity, these techniques serve to prime a student for learning by reviewing, highlighting and sharing relevant material from prior lessons and personal student knowledge. During an activity, these strategies can be used to organize material being taught for easier student recollection and understanding. Following an activity, these educational strategies summarize the learning experience in a useful, learner-friendly format. In all cases, use of these strategies enhances a student's learning experience by reinforcing the need for active, lasting information processing.

- **KWL Charts**
  These three-column charts are used to record "What students already **k**now," "What they **w**ant to know" and "What they have **l**earned." The first two columns can be used as a preparatory strategy, and the last column can be used as a summarizing technique.
- **Concept Mapping**
  This technique serves to demonstrate a student's ability to organize knowledge. In this technique, students must be able to demonstrate true understanding of the relationship between various concepts.
- **Venn Diagrams**
  These diagrams can be used to compare and contrast different topics from classroom material. This technique can be used to demonstrate interrelation between information from different lessons.
- **Jigsaw/Reciprocal Teaching**
  This strategy allows students to become experts on certain topics/aspects of a lesson and share their knowledge. Allowing students to teach sections of a lesson to the rest of the class can be a powerful tool. It enhances student learning and can build student confidence.
- **Reflective Teaching**
  Teachers, remember to use your "Activity Notes" section provided in the Toobeez activity workbooks. These notes can be used to improve future lesson implementation by highlighting any encountered challenges.

ACTIVITY 1 YOU SANK MY BATTLESHAPE

Objectives

- Familiarize students with the Cartesian plane system to plot values on a graph
- Identify X and Y axes and +/- quadrant values
- Identify different geometric shapes
- Aid students in understanding the mathematical significance of the Cartesian plane
- Use predicting skills to determine the positions of opponent's "battleshapes"
- Work cooperatively with others and in a teambuilding style

Preparation

Setup Time: 30 minutes
Materials:

1 Toobeez set
4 dry-erase markers
4 pieces of oak tag or poster paper
2 different-colored markers per student
graph paper

Activity Plan

Time: 2 hours
Space: Medium
Instruction: Whole class, pairs and individual

Character Focus

Teamwork/Cooperation & Communication

The Challenge

Each group must try to "sink" the opposing team's Toobeez battleshapes using knowledge of the Cartesian plane system.

Battleshape

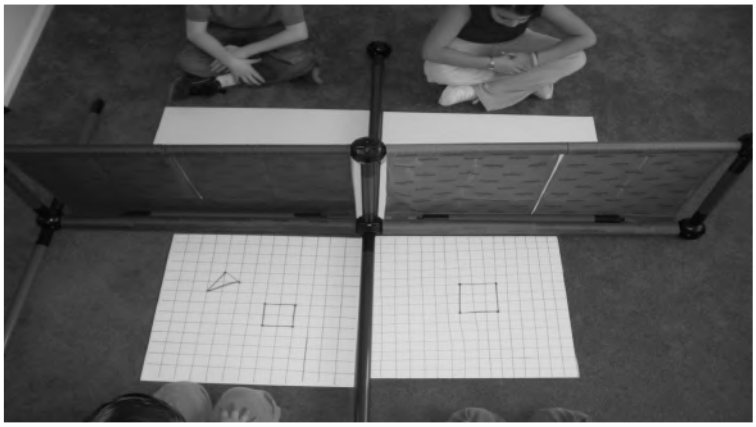
Activity Setup

1. Construct a cross shape using four of the 36" Toobeez tubes with one sphere in the center joining the four tubes. Also connect a sphere to the end of each 36" Toobeez. These will serve as the X and Y axes for the Cartesian plane playing board. **Teacher Note**: The northern and eastern tube should be the same color. Likewise, the southern and western tubes should also match in color. This serves to reinforce the negative and positive values along the X and Y axes of the playing board.

2. Place one 36" x 36" square of chart paper or oak tag in each quadrant of the constructed Cartesian plane. Using a meter stick and marker, students should draw a straight line along the X axis every 8.6 centimeters. These lines should be parallel to each other and extend the length of the chart paper in each quadrant.
3. Have the students note the appropriate positive or negative values of each line along the X axis on the chart paper.
4. Repeat Steps 2 and 3 for the Y axis. The result is a 2-D grid pattern.

Battleshape

5. Using Toobeez pieces and curtains, have students construct a dividing structure, as shown, equal in length to the X axis. This partition is constructed along the X-axis by placing two Toobeez curtains vertically on each side and then rolling up the curtain from the bottom to tighten. This partition will divide the Toobeez Cartesian plane playing board in half. You are now ready to play Toobeez Battleshapes.



6. In addition to using the Toobeez game board, each student should be provided with a sheet of graph paper.

Helpful Hints!

Be sure to review these tips prior to beginning the activity, and if necessary, share reminders with the group during the activity.

- Before the activity, have students draw a Cartesian plane model on graph paper to ensure understanding
- Review the difference between positive and negative numbers. It is helpful to review mathematical operations using positive and negative integers

Battleshape

Activity Instructions

1. After dividing students into teams, each team should construct three two-dimensional geometric models using Toobeez (one triangle, one square, and one pentagon) to serve as their battleshapes.
2. Read aloud the following Activity Challenge Box to the group.

> **Challenge**: Each group must try to "sink" the opposing team's Toobeez battleshapes using knowledge of the Cartesian plane system.

3. Using markers, each team should secretly plot their three battleshapes on their side of the Toobeez Cartesian plane playing board curtain. Each sphere of the models constructed in Step 1 should serve as a point on the Toobeez Cartesian plane.
4. After plotting all three battleshapes, each team should also write the coordinates of each point on their team's playing board for their own easy reference during play.
5. Taking turns, teams should call out coordinates in the appropriate manner (X,Y), and be mindful of positive and negative values.
6. If a team calls out a point that corresponds to the plotted point of an opposing team's battleshape, it counts as a hit. Whenever a team guesses all the plotted points of an opposing team's shape, that shape is considered to have been sunk. Guesses that do not coordinate with the plot points are misses but can aid in the process of elimination to guess the correct coordinates.
7. From the coordinates they call, team members should record their hits and misses on both the playing board and their personal plane handouts to prevent recalling the same coordinates.
8. The first team to sink all the opposing team's battleshapes by identifying their plot points wins!
9. After the activity, circle up the group and ask them the following question: "How does the Cartesian plane coordinate system work?"
10. Finally, move to the "Activity Discussion and Processing" section of the activity.

Battleshape

Assessment

- Assign drawings of geometric figures with specific coordinate points. Have the students properly name the coordinates of all vertices
- Assign each student three more advanced geometric shapes. Pair these students off and let them play "Battleshapes" at their desks using Cartesian planes drawn on graph paper

Activity Discussion and Processing

To close the lesson, end with a group discussion about what was learned during the activity. Circle up the group and work through the following questions. If possible, record the group's responses on flip chart paper so all comments are displayed.

- How is a coordinate system useful in math?
- How did the coordinate system and recording "hits and misses" aid the team in locating hidden battleshapes?
- Why/how can a coordinate system be useful in everyday life?
- How can the Cartesian plane two-dimensional system be enhanced? (Advanced students can discuss the use of a three-dimensional system including a Z-axis)
- Why were cooperation and communication vital to learning the Cartesian coordinate system in this activity?

Here are some additional topics for class discussion:

- The construction of the playing board and grid
- The building and identification of shapes with classmates
- The team effort in understanding the Cartesian plane coordinate system

Battleshape

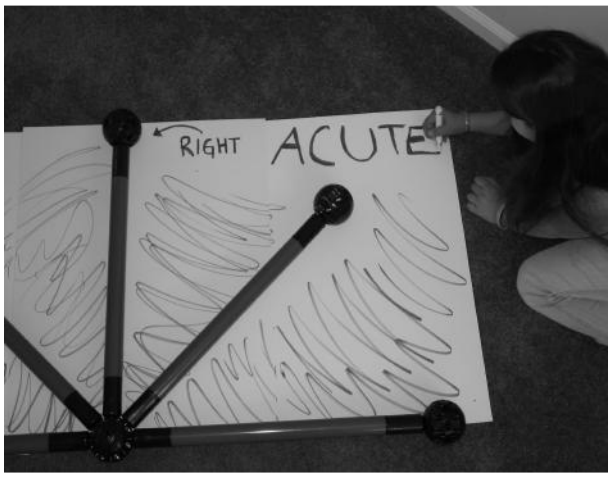
Activity Variations

1. **A different view.**
Increase the sizes of the battleshapes used by each team. For example, use hexagons and octagons.
2. **Additional challenge.**
Make four student teams and subdivide the Toobeez Cartesian plane playing board, making each quadrant a playing field.
3. **Extension/Follow up.**
Using a fresh Toobeez Cartesian plane playing board, guide the class in identifying coordinate points of shapes drawn on the Toobeez Cartesian plane.

Activity Notes

ACTIVITY 2

PROTRACTOR PRACTICE



Objectives

- Familiarize students with the protractor
- Enable students to define and classify obtuse, acute and right angles
- Practice drawing examples of the three angle types
- Aid students in learning measurement skills by using a protractor
- Work cooperatively with others in a teambuilding style

Preparation

Setup Time: 15 minutes
Materials:

1 Toobeez set per 1 - 3 groups
1 piece of chart paper per group
3 different-colored markers per group
1 sheet of graph paper per student
1 protractor per student
1 pen or pencil

Activity Plan

Time: 35 - 45 minutes
Space: Medium
Instruction: Whole class, pairs and individual

Character Focus

Teamwork/Cooperation & Communication

The Challenge

The group should be able to use a protractor to identify qualities of angles.

Protractor Practice

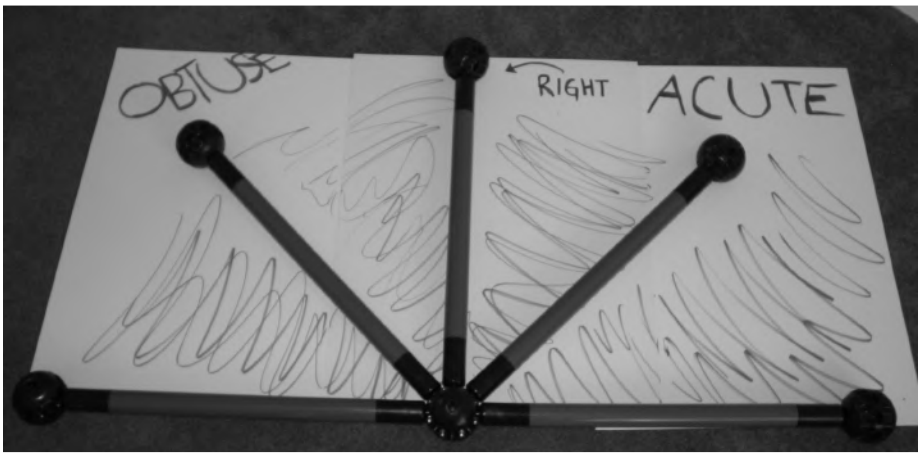
Activity Setup

1. Teachers should tape chart paper to the chalkboard. Students should lay their chart paper on the desk so it is visible to the group.
2. Connect two 24" Toobeez tubes into a straight line along the multi-holed equator of a sphere. This will serve as the 0º/180º line of the Toobeez protractor model. (*If enough Toobeez sets are not available, three models can be made per Toobeez set. The teacher model can be modified by using 36" tubes, and student groups can construct models using either 24" or 16" tubes.)

3. Insert a 24" Toobeez tube into the center sphere along the multi-holed equator so that it is at a right angle with the tubes of the 0º/180º line. This will serve as the 90º line of the Toobeez protractor model.

4. Insert two additional 24" Toobeez, one on either side of the 90º line, so that both are midway between the 90º and 0º/180º line.

5. Teachers should hold their protractor model up to the center of the chart paper and have students do the same in their groups.
6. Using a red marker, color the chart paper to the right of the 90º tube. Write the word "acute" (<90º) above this area.
7. Using a green marker, write the word "right" above the 90º tube.
8. Using a blue marker, color the chart paper to the left of the 90º tube. Write the word "obtuse" (>90º) above this area.



Protractor Practice

Helpful Hints!

Be sure to review these tips prior to beginning the activity, and if necessary, share reminders with the group during the activity.

- Teachers should often remind students that the Toobeez protractor is a model to compare with their own protractors
- Remind students that the tubes of the Toobeez model are representative of particular angle measurements (for example, center tube = 90º, middle tubes = 45º and 135º)
- Teachers may want to use an erasable marker to write the measurement values on the model tubes

See page 6 for available training options!

Activity Instructions

1. Divide the students into groups.
2. Read aloud the following Activity Challenge Box to the group.

Challenge: The group should be able to use a protractor to identify qualities of angles.

3. Teachers should begin with a mini-lesson on measuring angles and protractor use. **Hint:** Use the Toobeez protractor model as an educational prop in the mini-lesson.
4. Using the remaining Toobeez pieces, each group should build a model of each type of angle and classify it.
5. At their desks, each student should draw three examples of obtuse and acute angles.
6. Using their protractors, students should measure their hand-drawn angles.
7. After the activity, circle up the group and ask them the following questions: "How is a protractor used to identify types of angles?"
8. Finally, move to the "Activity Discussion and Processing" section of the activity.

Assessment

- Using their protractors, have students draw angles of specific measurements
- Quiz students using angle models made from Toobeez. Have students classify these models as obtuse, acute and right angles

Activity Discussion and Processing

To close the lesson, end with a group discussion about what was learned during the activity. Circle up the group, and work through the following questions. If possible, record the group's responses on flip chart paper so all comments are displayed.

- Describe the difference between obtuse, acute and right angles
- How can knowing the type of angle assist you in describing the angle?
- In what real world scenarios would knowledge of angles be useful?
- How did working as a group to build and classify angles enhance your understanding?

Here are some additional topics for discussion:

- The procedure for using a protractor
- The classification system of angles
- The building and drawing of angles to meet certain specifications
- The team effort used in building the protractor and angle models

Protractor Practice

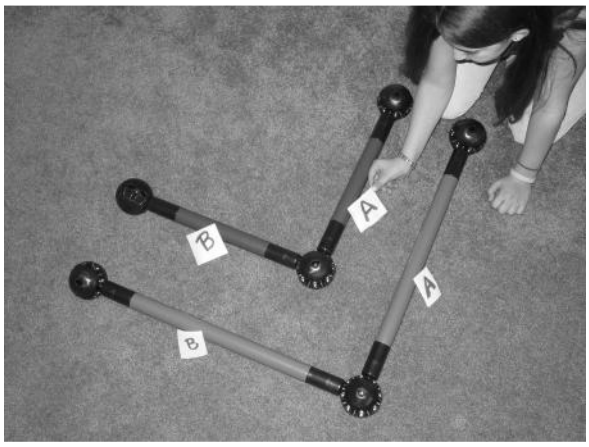
Activity Variations

1. **A different view.**
   Insert eight 24" Toobeez tubes into a sphere along the multi-holed equator. Present this to students as the interior model of a circle. Challenge the class to deduce from the model how many degrees are in a complete circle.
2. **Extension/Follow up.**
   At their desks, have students draw several examples of some geometric shapes (for example, triangle, square and rectangle). Then have the students measure and record the angular measurements of these shapes. Challenge the students to identify any observed mathematical rules from their findings (for example, all the angles in a triangle add up to 180º).

Activity Notes

ACTIVITY 3

PYTHAGOREAN PUZZLER



Objectives

- Introduce students to the Pythagorean theorem
- Aid students in manipulating and solving Pythagorean equations
- Use predicting skills to determine the lengths of the various sides of right triangles
- Aid students in understanding the mathematical significance of the Pythagorean theorem
- Work cooperatively with others to understand mathematical concepts in a teambuilding style

Preparation

Setup Time: 10 minutes
Materials:

1 Toobeez set
1 meter stick
1 sheet of graph paper per student
1 pen/pencil per student

Activity Plan

Time: 40 - 50 minutes
Space: Medium
Instruction: Whole class, pairs and individual

Character Focus

Teamwork/Cooperation & Communication

The Challenge

Each group will construct right triangles using Toobeez and learn to predict side lengths of triangles using the Pythagorean theorem.

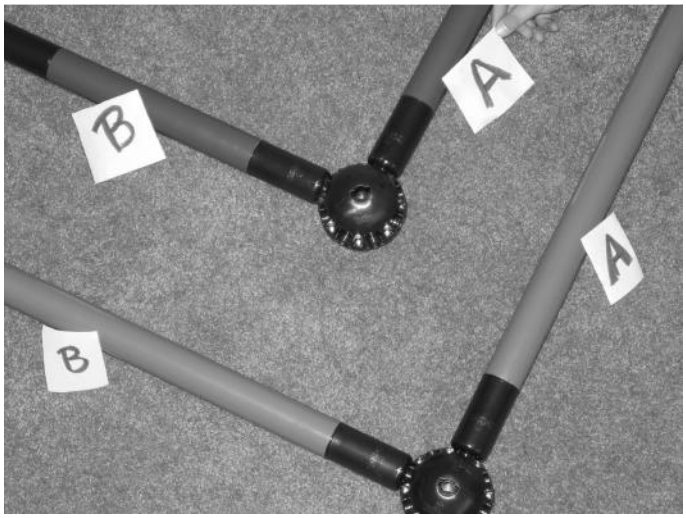
Pythagorean Puzzler

Activity Setup

1. Have each group build two right angles using the following Toobeez tubes combinations:
    a. Two 16" Toobeez tubes
    b. Two 24" Toobeez tubes

2. Students should lay their chart paper on the desk so it is visible to the group.

3. Tape a note card to each tube. Label one tube in each triangle "A" and label the other tube "B." These will serve as the right angles in the groups' triangles.



Pythagorean Puzzler

Helpful Hints!

Be sure to review these tips prior to beginning the activity, and if necessary, share reminders with the group during the activity.

- The Pythagorean theorem states, in algebraic terms, $a^2 + b^2 = c^2$ where c is the hypotenuse and a and b are the sides of a right triangle
- Make sure to remind students to measure the sides of Toobeez triangles using similar points on each vertex (sphere). Failure to do so can result in inconsistent results. **Hint:** Have students measure from the top center hole of each sphere
- Be sure to practice building these triangles in advance of the lesson
- Have students practice measuring with a meter stick

Activity Instructions

1. After dividing the students into teams, a mini-lesson should be presented to the class on the Pythagorean theorem. Be sure to review techniques for solving the equation.
2. Read aloud the following Activity Challenge Box to the group.

Challenge: Each group will construct right triangles using Toobeez and learn to predict side lengths of triangles using the Pythagorean theorem.

3. Using the meter stick, groups should measure and record the length of the two sides of each angle built during the setup phase of the activity.
4. Groups should use the Pythagorean theorem to solve the length of the hypotenuse using the side lengths measured in Step 3.
5. As a group, students should use a meter stick and their answer from Step 4 to deduce which tube length is required to build a right triangle from the angle models. A consensus should be reached in the student groups.
6. Students should try to then build a right triangle using a tube of the agreed upon length.
7. After the activity, circle up the group and ask them the following question: "How can you use the Pythagorean theorem to identify which tube length was required to build a right triangle?"
8. Finally, move to the "Activity Discussion and Processing" section of the activity.

Assessment

- Have students draw right triangles using a protractor and prove the Pythagorean theorem using these models
- Build a right triangle using Toobeez and remove one of the side legs, leaving the hypotenuse and other leg intact. Give this model to student groups and have them solve for the length of the missing leg

See page 6 for available training options!

Activity Discussion and Processing

To close the lesson, end with a group discussion about what was learned during the activity. Circle up the group, and work through the following questions. If possible, record the group's responses on flip chart paper so all comments are displayed.

- Do all triangles observe the Pythagorean theorem? (Have students draw right and non-right triangles to disprove this assumption)
- Are there relationships in the side lengths of other types of geometric figures?
- Why/how could the Pythagorean theorem be useful in everyday life?
- How did using the Pythagorean theorem enhance communication between students?

Here are some additional topics for discussion:

- The mathematical technique for solving the Pythagorean equation
- The relationship between the legs and the hypotenuse
- The team effort in solving the Pythagorean puzzles

Activity Variations

1. **A more complex puzzle.**
   Students should build an initial right angle using two 36" Toobeez tubes. Have students solve for the hypotenuse and build a right triangle. **Hint:** The hypotenuse required consists of two 24" tubes with a sphere in the center.
2. **Extension/Follow up.**
   Have students draw right triangles, collect length and angular measurements, and note any observable relationships between the two sets of variables.

Activity Notes

ACTIVITY 4

PROVING PROBABILITY

Objectives

- Introduce and demonstrate the concept of probability
- Use predicting skills to determine the validity of probability theory
- Conduct an experiment to demonstrate the real world application of a mathematical concept
- Work cooperatively to prepare experimental materials, collect data and draw conclusions

Preparation

Setup Time: 15 minutes
Materials:

1 Toobeez set
1 dry-erase marker
notebook
pen/pencil

Activity Plan

Time: 40 - 50 minutes
Space: Medium
Instruction: Whole class, pairs and individual

Character Focus

Teamwork, Cooperative Problem Solving & Perseverance

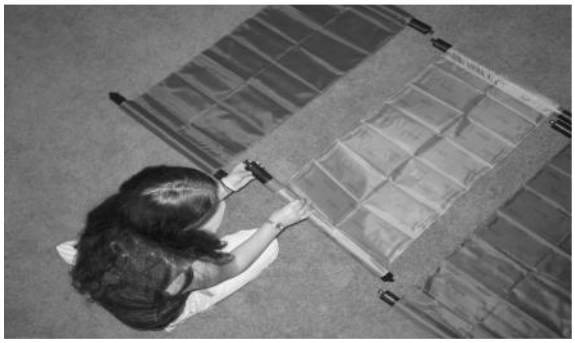
The Challenge

Each group must prove the validity of probability using the ball bucket experiment.

Proving Probability

Activity Setup

1. After dividing the students into groups, each group should use the following steps to build the ball bucket.
2. Insert a 24" Toobeez tube into both ends of all four curtains.



3. Attach spheres to both ends of the 24" tubes in two of the curtains from Step 2.

4. Using four 36" Toobeez tubes, construct two rectangular panels with the two curtains from Step 3.

5. Connect the two panels from Step 4 with the remaining two curtains from Step 2.
6. Stand the resulting box up with one open end facing the floor (see right). This will serve as the ball bucket for the experiment.

7. Using the remaining spheres in the Toobeez set and an erasable marker, students should write one letter (A, B or C) on each sphere or on a label taped to each sphere. **Teacher Note:** Be sure to have students record the total number of each type of lettered sphere (see below).

See page 6 for available training options!

8. Toss the lettered spheres into the ball bucket.

Proving Probability

Helpful Hints!

Be sure to review these tips prior to beginning the activity, and if necessary, share reminders with the group during the activity.

- Probability is an approximation of the likelihood of an event or outcome
- Probability equals the number of ways an event can occur or the number of possible outcomes
- Probability is always a number between 0 and 1, inclusive. The closer that number is to 1, the more likely an event is to occur

Activity Instructions

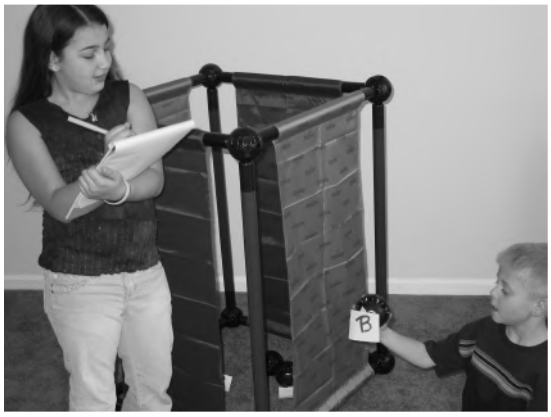
1. After setting up, divide the students into groups.
2. Begin with a mini-lesson on probability.
3. Read aloud the following Activity Challenge Box to the group.

> **Challenge**: Each group must prove the validity of probability using the ball bucket experiment.

4. As a group, students should define the three possible outcomes (A, B or C) for drawing a ball from the bucket. **Teacher Note:** This should be a student-led discussion. Allow students to self discover the possible outcomes.
5. The students should calculate the probability of each event using the following equations:
    1. P(A)= # of ways to choose "A"/total # of spheres
    2. P(B)= # of ways to choose "B"/total # of spheres
    3. P(C)= # of ways to choose "C"/total # of spheres

   Teacher Note: Students should be invited to try and solve the probability of each event using the master probability equation provided during the mini-lesson.
6. Groups should conduct an experiment by removing one sphere, recording the letter, and replacing the sphere in the bucket. Students should repeat this procedure at least 20 times. **Teacher Note:** The more attempts conducted, the more accurate the calculated probabilities.

Proving Probability



7. Using the equations in Step 5, students should calculate the actual frequency of each event.
8. Students should compare the predicted probability and actual frequency of each event. Each group should record any interesting observations.
9. Each group should compare and share their results with the entire class.
10. After the activity, circle up the group and ask them the following question: "Were the actual experimental frequencies similar to the calculated probabilities? What does this information tell you about probability?"
11. Finally, move to the "Activity Discussion and Processing" section of the activity.

Assessment

- Lead the class in a group discussion as to why the actual frequency and calculated probability may differ slightly
- Explore with the class how the accuracy of the experimental results could be enhanced (**Hint:** increasing the number of trials from 20 to 40)

Activity Discussion and Processing

To close the lesson, end with a group discussion about what was learned during the activity. Circle up the group, and work through the following questions. If possible, record the group's responses on flip chart paper so all comments are displayed.

- Why is probability an important concept?
- How would increasing the number of attempts in the experiment affect the relationship between the actual frequency and the calculated probability of an event?
- Why/how can probability be useful in everyday life?
- Why are cooperation and perseverance crucial elements in successfully completing an experiment?

Here are some additional topics for discussion:

- The concept of probability as a predictive tool
- The use of experimental techniques to prove a mathematical theory
- The team effort used in conducting the experiment and collecting data

Activity Variations

1. **Increasing accuracy.**
   Have the students conduct the same experiment, but with 40 trials rather than 20. Let the students compare this result to the original experimental result and the calculated probabilities.
2. **Additional challenge.**
   Have students repeat the activity with four types of lettered spheres (A, B, C & D).
3. **Extension/Follow up.**
   Have the students conduct the experiment by removing only ten spheres from the ball bucket without replacing them. Determine the effect on the calculated probability.

Activity Notes

ACTIVITY 5

MATH HOT POTATO

Objectives

- Provide a structured, interactive game to review learned materials
- Challenge students to answer potential test questions under time-sensitive conditions
- Foster interactive cooperation in a competitive environment
- Promote student-to-student instruction in more complex problem solving
- Work cooperatively with others in a teambuilding style

Preparation

Setup Time: 20 minutes
Materials:

1 Toobeez set per group
1 package of index cards
1 stopwatch
1 notebook per student
1 pen/pencil per student

Activity Plan

Time: 50 - 60 minutes
Space: Medium
Instruction: Whole class, pairs and individual

Character Focus

Teamwork, Cooperative Learning, Respect & Trust

The Challenge

The student groups should attempt to answer all the sample problems in a timely manner. Likewise, they should seek to learn from other groups by challenging them to answer problems their group finds too challenging.

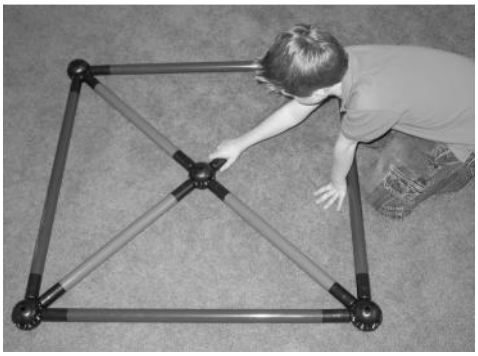
Math Hot Potato

Activity Setup

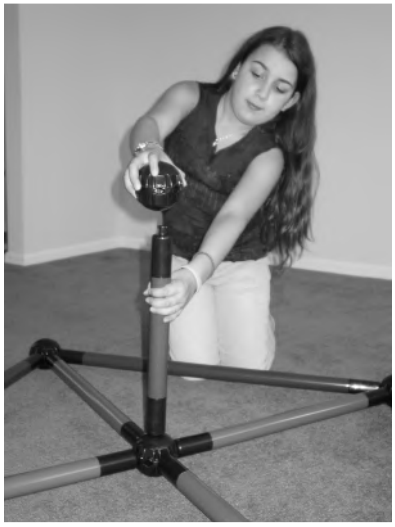
1. Connect four 36" Toobeez tubes into a square formation so that the multi-holed equator of each sphere is in a horizontal position.

2. Insert a 24" Toobeez tube into the interior of the square using the vacant hole between each 36" Toobeez tube from Step 2. This should form a square with four 24" Toobeez tubes converging in the center.

3. Connect the four converging 24" tubes in the center with a sphere. This will serve as the base for the Toobeez hot potato apparatus.



4. Insert a 16" Toobeez tube in the top hole of the center sphere from Step 4 so that it is vertical. Attach another sphere to the top of this tube. Insert a 16" Toobeez tube into the top sphere so that it is parallel to the ground.



5. Attach a sphere to the end of the 16" Toobeez tube from Step 6. This will serve as the hot potato to be passed during game play.
6. Depending on the number of index cards available, teachers should prepare the game cards in advance according to the following guidelines:
    - A majority of cards should have a sample test question based on the current learning topic (with only one problem per card)
    - A few cards should be marked with the word "Skip" or "Reverse"
7. This playing deck should be placed face down in the center of the Toobeez hot potato apparatus so that it is accessible from all four sides.

Math Hot Potato

Helpful Hints!

Be sure to review these tips prior to beginning the activity, and if necessary, share reminders with the group during the activity.

- As this is a test prep activity, students should not be allowed to refer to notes
- If a group finds a problem too difficult, they can pass (see "Activity Instructions"). However, they should be encouraged to use this opportunity to learn from other groups that are able to solve the problem
- Teachers should encourage students to work within their group in a teambuilding manner to solve problems

See page 6 for available training options!

Activity Instructions

1. Divide the class into four groups. **Teacher Note:** Each group should include students of varying mathematical skill levels.
2. Read aloud the following Activity Challenge Box to the group.

Challenge: The student groups should attempt to answer all the sample problems in a timely manner. Likewise, they should seek to learn by challenging other groups to answer problems their group finds difficult.

3. Teachers should begin by outlining the following procedure for play and rules. Teachers are responsible for timing the rounds.
4. Each round will last four minutes. **Teacher Note:** This can be changed depending on the skill level or the topic under review.
5. A group should be chosen to go first, and the game proceeds to the left.
6. On their turn, one member from the group must always be holding the end sphere extending from the center Toobeez tube. This is the hot potato.

Math Hot Potato

7. While the student is holding the hot potato, another student should draw one card from the top of the playing deck and read the problem or directions listed on the card for the whole group to hear.
8. Depending on the card type drawn, one of the following should be done:
    - Sample problem card: The group must solve the problem to pass the hot potato to the next group
    - Skip: The hot potato should be passed to skip the next group
    - Reverse: The hot potato should be passed in the direction opposite to current game play
9. "Skip" or "Reverse" cards should be set aside after use.
10. If a team answers a problem card correctly, they should keep that card and pass the hot potato to the next group.
11. If the team provides an incorrect answer, that problem card should be returned to the bottom of the center deck and the hot potato should be passed to the next group.
12. Challenging: If a team decides a problem card is too difficult, they can choose to challenge any other group.
    - If the challenged group is correct, they keep that card and go again. The hot potato is passed off to the challenged team and game play continues in the same direction
    - If the challenged group is incorrect, the problem card is returned to the challenging team. They keep that card and can draw again from the game deck
13. This game play proceeds for four minutes.
14. Scoring:
    - Add one point for each correctly answered problem card collected
    - Take away two points for the group holding the hot potato at the end of the round
    - The team with the most points wins!

15. After the activity, gather the class and pose the following questions: "How did this exercise help you to review the test material?" and "What problems in the review were the most difficult?"
16. Finally, move to the "Activity Discussion and Processing" section of the activity.

Assessment

- Play a round of hot potato immediately before a test/quiz to review. Give bonus test/quiz points to the winning team
- Conduct a whole class review of any problem cards that were unanswered during a challenge
- If a difficult problem is answered by a group during a challenge, have the group that solved the problem present the problem to the class

Activity Discussion and Processing

To close the lesson, end with a group discussion about what was learned during the activity. Circle up the group, and work through the following questions. If possible, record the group's responses on flip chart paper so all comments are displayed.

- How is competing to solve problems in a time-sensitive manner helpful in test preparation?
- Did any problems answered correctly by other groups seem difficult? If so, review these problems
- Why is it useful to observe how others solve problems that you did not understand?
- Why are communication and trust crucial in group problem solving?

Here are some additional topics for discussion:

- Highlight and conduct a review of the current topic being studied
- The ability to learn by observing others solve problems
- The team effort used in decision making and problem solving

Math Hot Potato

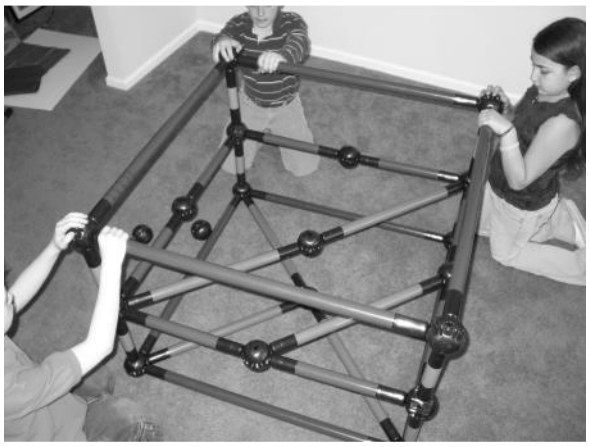
Activity Variations

1. **Challenging each other.**
Allow students to design the problem deck using homework problems which they found difficult.
2. **Skill level assessment.**
While designing the game deck, assign differing point values to problem cards based upon the skill level required.

Activity Notes

ACTIVITY 6

CONTRACTOR'S CRISIS



Objectives

- Hone students' problem-solving skills
- Reinforce the principle of area
- Introduce a real-world application of area
- Practice measurement skills
- Strongly reinforce cooperative teambuilding by seeking to meet prescribed requirements

Preparation

Setup Time: 15 minutes
Materials:
1 Toobeez set
1 set of instructions per student
1 meter stick
1 notebook per student
pen or pencil

Activity Plan

Time: 1 - 2 hours
Space: Medium
Instruction: Whole class and individual

Character Focus

Teamwork/Cooperation, Interactive Problem Solving & Perseverance

The Challenge

The class should be able to build a three-story house model according to specified requirements and calculate the money required to wallpaper and carpet all the rooms.

Contractor's Crisis

Activity Setup

1. As a homework assignment the prior evening, students should be asked to research in newspapers or online for wallpaper and carpet pricing that appeals to them.
2. To provide students with a starting point, teachers should construct a square using four 36" Toobeez tubes. **Teacher Note:** The multi-holed equator of the spheres should be in the horizontal position at each corner.

3. Teachers should prepare the following blueprint instructions in advance and make enough copies for one per student.
    - Each floor of the house is separated by 11" tubes
    - There must be at least two isosceles triangular rooms on the first floor
    - There must be two right-triangle-shaped rooms and one parallelogram-shaped room on the second floor
    - The top floor should consist of one large square roof garden with no walls required
    - All walls and floors must be covered with wallpaper and appropriate floor surface coverings

Contractor's Crisis Solution for teachers: (There are other potential solutions)

- **Between floors:** The eight 11" Toobeez tubes are used to separate the three floors (as pictured above)

(Solution is continued on the next page.)

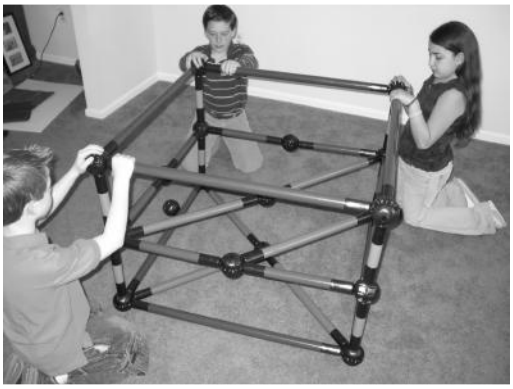
Contractor's Crisis

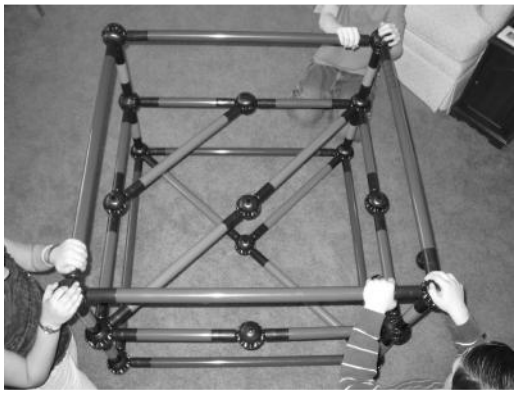
- **First floor:** Insert three 24" Toobeez tubes into the corner spheres of the foundation square. Connect them together in the center of the sphere using a sphere. **Teacher Note:** This should create two isosceles triangles and one right triangle within the square foundation

- **Second floor:** The foundation for the second floor should consist of eight 16" Toobeez tubes, with two per side separated by a sphere. The center sphere of one side should be connected to the center sphere of an adjacent side by a 24" Toobeez tube. In addition, parallel to this 24" tube, two additional 24" tubes joined by a sphere in the center should be inserted so that they connect two corners of the second floor model. This should produce a center room shaped like a parallelogram separating two right triangular rooms

(Solution is continued on the next page.)

- **Third floor:** A square can be made of the remaining four 36" Toobeez tubes. No walls are required for the roof garden





Helpful Hints!

Be sure to review these tips prior to beginning the activity, and if necessary, share reminders with the group during the activity.

- Teachers should remind students to refer to the provided building blueprint often during the construction process
- Review the equations for calculating the area of various geometric shapes
- Remind students there is more than one potential solution

Activity Instructions

1. Read aloud the following Activity Challenge Box to the group.

> **Challenge**: The class should be able to build a three-story house model according to specified requirements and calculate the amount of money required to wallpaper and carpet all the rooms.

2. Teachers should share the following scenario with the class:
   - You have been hired as contractors to build a house with the requirements listed in the building blueprint directives. Using the Toobeez set, build a model of the described house.

3. Teachers should inform the class that the provided structure from the "Activity Setup" will serve as the foundation for the first floor. No floor can be wider than the first floor.
4. Appoint two students, preferably more tactile learners, as builders to physically construct the model following cues provided by the class.
5. In an organized manner, students should be encouraged to make suggestions for the builders to meet the following blueprints. **Teacher Note:** Reinforce that all opinions are equally valuable.
6. After successfully building the blueprint model, have the class draw a floor plan layout for each floor in their notebooks. Teachers should also assign numbers to each room for easy identification and have students copy these numbers. **Teacher Note:** Remind students that there are three first floor rooms, three second floor rooms, and only one roof garden on the third floor that has no walls.
7. One student, the interior decorator, should measure, in inches, the floor dimensions area of each room and the outermost wall dimensions of the house model. **Teacher Note:** As there are no room dividing walls, wallpaper estimates will only be done on the outermost walls of the model house.
8. Students should be provided with a model conversion rate and informed that each inch in the house model equals 1 foot. (for example, 8.6 inches = 8.6 feet.)
9. Students should convert inch measurements into feet.
10. At their desks, students should calculate the floor area of each room and the area of each outermost wall in the model house.
11. Using their research homework on wallpaper and floor covering prices, students should calculate the amount of money required to cover the floor in each room and the amount required to cover each outermost wall.
12. Students should add all these figures to calculate the entire cost for floor treatment and wallpaper.
13. After the activity, gather the class together and ask the following question: "How can calculating the area of room surfaces be useful in house building and decorating?"
14. Finally, move to the "Activity Discussion and Processing" section of the activity.

Assessment

- Have the students build various geometric shapes with Toobeez, take their measurements, and calculate the area of each example
- Change the model conversion rate so that each inch measured is equivalent to 2 feet. (for example, 8.6 inches = 17.2 feet) Then, have students recalculate area and materials cost for the floors and walls

Activity Discussion and Processing

To close the lesson, end with a group discussion about what was learned during the activity. Circle up the group, and work through the following questions. If possible, record the group's responses on flip chart paper so all comments are displayed.

- In calculating the surface area of a cube-shaped room, how many surface areas must you calculate to determine the surface area of the room? **Teacher Note:** Tricky one! Only one: They are all identical. This is an excellent extension question to discuss volume
- If you owned a house, why might it be important to calculate room dimensions before decorating?
- What additional real-world scenarios require the ability to calculate the area of various geometric shapes?
- What steps in this activity require perseverance and cooperation?

Here are some additional topics for discussion:

- The use of spatial reasoning to meet the blueprint's specifications
- The use of dimension measurements to calculate area
- The use of spatial calculation to determine the cost of materials
- The team effort used in constructing the house model

See page 6 for available training options!

Activity Variations

1. A more crowded house.
Challenge the class to build a three-story house with more rooms than laid out in the original blueprint. Have them recalculate all the areas and materials cost and determine the effect on cost.

2. A more detailed design.
Review drawing scaled models. Split the class into groups and assign each group a particular room within the house model. On graph paper, help students draw a scaled floor model of their assigned room. For homework, students are to research room furnishings, both dimensions and cost. They should verify these furnishings can fit into their assigned room by drawing scaled representations of these furnishings within the drawn floor plan. Finally, students should calculate the total cost for decorating the room.

3. A 3-D view.
Review drawing scaled models. Assign each student a room to decorate. Aid students in drawing scaled models of the floor surface for each room and the outermost walls associated with each room. Inform students that they must also imagine that their room is surrounded by walls on all sides with a 6'5" doorway on one of the inner-facing walls. For homework, have students calculate the surface area for the floor and each wall. Using the prices of their wallpaper and floor treatment, have them also calculate the total cost of covering the walls and floor. Finally, have them calculate the volume of their assigned room.

Activity Notes

ACTIVITY 7

POLYGONS AND POLYHEDRA

Objectives

- Introduce students to 2-D and 3-D geometric figures
- Define polygons and polyhedra
- Learn nomenclature associated with 2-D and 3-D geometric figures
- Observe and identify features characteristic of various geometric shapes
- Work cooperatively with others in a teambuilding style

Preparation

Setup Time: 5 minutes
Mini-Lesson Time: 15 – 20 minutes
Materials:
1 Toobeez set
1 piece of chart paper per group
graph paper
pen or pencil

Activity Plan

Time: 1 hour
Space: Medium
Instruction: Whole class and individual

Character Focus

Teamwork/Cooperation & Communication

The Challenge

Students should be able to identify and name various polygons and polyhedra.

Polygons and Polyhedra

Activity Setup

1. Teachers should build a 2-D triangle and a cube from Toobeez tubes (as pictured).

Activity Mini-Lesson

1. Ask students to evaluate the two shapes and record observations in their notebooks. Lead a discussion which allows students to share their observations about the two figures. **Teacher Note:** The crucial observation points are as follows:
    - One figure is 2-D and the other is 3-D
    - None of the lines in either figure intersect
2. After the discussion in Step 1, the teacher should identify the triangle as a polygon and the cube as a polyhedron. Based upon the discussion, the teacher should guide the class in a discussion to define the polygon and polyhedron. Any definitions that encompass the following concepts are acceptable:
    - Polygon: A 2-D figure made up of a closed chain of points linked together by non-intersecting straight lines
    - Polyhedron: A 3-D figure made up of a closed chain of points linked by non-intersecting straight lines
3. Remove one of the Toobeez tubes from the triangle models. Ask the class, "Is this a polygon?" Follow up with, "If it is not, explain why?" and "What quality does the polygon have that distinguishes it from a polyhedron?"
4. After establishing the definitions, remove one tube from the cube model constructed in the setup. Ask the class: "Is this a polyhedron?" Follow up with: "If it is not, explain why?" and "Does removal of the one Toobeez tube convert it into a polygon? If not, why?"
5. Have students record their conclusions and observations from this discussion in their notebooks.
6. Teachers should inform students that different types of polygons and polyhedra have different names.
7. Students should brainstorm in preparation for the activity to identify what qualities would best serve in classifying these two types of geometric figures.

Polygons and Polyhedra

Helpful Hints!

Be sure to review these tips prior to beginning the activity, and if necessary, share reminders with the group during the activity.

- Teachers should often remind students of the two definitions established during the "Activity Setup" discussion
- The teacher should emphasize the differences between the two classes of geometric shapes
- Have students share their results for the potential parameters for classification from the discussion in the "Activity Setup." **Teacher Note:** Try to guide students in a discussion to eliminate incorrect hypotheses and to include any missed classification parameters

See page 6 for available training options!

Activity Instructions

1. Divide the students into groups.
2. Read aloud the following Activity Challenge Box to the group.

   Challenge: Students should be able to identify and name various polygons and polyhedra.

3. If accurate classification parameters for polygon and polyhedron were not arrived at through the setup discussion, the following parameters should be introduced:
   - Polygons are classified and named by the number of sides
   - Polyhedra are classified and named by the number of faces

   Teacher Note: To eliminate any confusion, review other classification parameters suggested by students and have the class discuss them.

4. After the parameters for classification and naming are established, the following table should be posted on the chalkboard for students to record:

Number of sides or faces	Prefix
3	tria
4	tetra
5	penta
6	hexa
7	septa
8	octa
9	nona
10	deca
12	dodeca
20	icosa
30	triaconta

Teacher Note: Point out the Greek origin of these prefixes.

5. After allowing students to review the chart in Step 4, point out that the number of sides in a polygon and the number of faces in a polyhedron the appropriate prefix is used in naming the figure.
6. Next, guide the students in the general rules for naming polygons and polyhedra.
    - Polygon - Use the appropriate prefix for the number of sides and add "–gon" to the end (for example, eight-sided polygon = octagon)
    - Polyhedron – Use Use the appropriate prefix for the number of faces and add "–hedron" to the end (for example, four-faced polyhedron = tertrahedron)

    Teacher Note: Be sure to point out some exceptions to this rule such as triangle and cube.
7. Using Toobeez, have each team build any random polygons, name them and record any additional observations about their polygons.
8. Have each group present their results to the class. Using their models, groups should explain their reasoning for naming the polygon.
9. Have the students break into their groups again and build various random polyhedra with Toobeez. Groups should then name the models and record any additional observations.
10. Have each group present their results to the class. Using their models, groups should explain their reasoning for naming the polyhedron.

11. After the activity, gather the class together and pose the following questions for discussion: "What are the differences between polygons and polyhedra? How is this difference reflected in the naming strategies for the two types of geometric figures?"
12. Finally, move to the "Activity Discussion and Processing" section of the activity.

Assessment

- On graph paper, have students draw various polygons, assign them names, and explain the name assignment
- On graph paper, have students draw various polyhedra, assign them names, and explain the name assignment

Activity Discussion and Processing

To close the lesson, end with a group discussion about what was learned during the activity. Circle up the group, and work through the following questions. If possible, record the group's responses on flip chart paper so all comments are displayed.

- What is the difference between the standard rules for naming polygons and polyhedra?
- Why is a foreign language used as the basis for naming polygons and polyhedra?
- Why is it important to have a systematic method for naming various geometrical figures?
- How are polygons and polyhedra useful in real world scenarios?
- What important aspect of communication does the naming system for polygons and polyhedra demonstrate?

Here are some additional topics for discussion:

- The difference between polygons and polyhedra
- The parameters for naming and classifying polygons and polyhedra
- The Greek-based system utilized in naming these figures
- The team effort used in building and naming geometric models

Activity Variations

1. **A spatial analysis.**

Have students measure the various dimensions of their Toobeez polygons and polyhedra. Have them calculate the area of the polygons and the volume for the polyhedra. Provide them with a listing of the appropriate equations. Ask them to explore why it is impossible to provide a volume for polygons. Also have them calculate the area of each face of a polygon (a cube is the easiest) and then calculate the sum of these areas. Explore why this sum is not equivalent to the calculated volume.

2. **Extension/Follow up.**

For homework, assign students various polygons to draw on graph paper. Instruct them to measure and record both side length and degrees of angles. Have them record any observations from their results. **Teacher Note:** Require students to draw four examples of each type of polygon assigned so that any mathematical qualities can be observed.

Activity Notes

ACTIVITY 8

A MATH RIDDLE: EULER'S FORMULA

Objectives

- Review the characteristics of a polyhedron
- Observe and record various qualities of different types of polyhedra
- Use deductive reasoning and observation to independently arrive at a mathematical formula
- Prove the validity of a universal mathematical formula governing polyhedra
- Work cooperatively in a deductive reasoning style with others to reach a conclusion

Preparation

Setup Time: 5 minutes
Mini-Lesson Time: 10 minutes
Materials:
1 Toobeez set per group
1 notebook per student
pen or pencil

Activity Plan

Time: 40 – 50 minutes
Space: Medium
Instruction: Whole class and small groups

Character Focus

Teamwork/Cooperation

The Challenge

Through systematic observation and deductive reasoning, the group should be able to define an equation that is valid for any convex polyhedron (Euler's formula).

Euler's Formula

Activity Setup

1. Teachers should build two Toobeez models, one of a polyhedron and one of a polygon.

Activity Mini-Lesson

1. Teachers should begin by presenting a Toobeez model of a polyhedron. **Teacher Note:** This should serve as a review. You can use Activity #7 "Polygons and Polyhedra" as an introduction to polyhedron theory
2. As a class, the students should be asked to recall the definition of a polyhedron and to highlight its unique properties. **Teacher Note:** It is a good idea to construct a Toobeez model of a polygon and ask the class to distinguish between the two types of figures.
3. After reviewing the definition of a polyhedron, lead the class in a review of the nomenclature of polyhedra. **Teacher Note:** Activity #7 provides an excellent introduction to this topic.
4. In addition to the number of faces, point out to students that a polyhedron can also be described by the nature of its angles.
    - Convex polyhedron: All the angles are less than 180º
    - Concave polyhedron: One or more of the angles is greater than 180º

    Teacher Note: Point out that it is easy to visually distinguish the two types of polyhedra by examining the angles. If the legs of an angle appear to face outward from the figure's center, the polyhedron is concave.
5. Inform the class that today's lesson will focus on convex polyhedra.
6. On a blank page in their notebooks, instruct students to draw a table with four columns. The four headings for these columns are: Name of Polyhedron, Faces, Edges and Vertices.

Euler's Formula

Helpful Hints!

Be sure to review these tips prior to beginning the activity, and if necessary, share reminders with the group during the activity.

- Teachers should remind students to use this exercise as a review of polyhedron nomenclature
- Reinforce the important differences that distinguish polygons and polyhedra
- Remind students that the formula they are trying to deduce only governs convex polyhedra

See page 6 for available training options!

Activity Instructions

1. Divide the students into groups.
2. Read aloud the following Activity Challenge Box to the group.

> **Challenge**: Through systematic observation and deductive reasoning, the group should be able to define an equation that is valid for any convex polyhedron (Euler's formula).

3. Using Toobeez, groups should be directed to build four different types of convex polyhedra. **Teacher Note:** Stress that the polyhedron must be convex and fulfill the definition of a polyhedron.
4. After building each model, the group should observe and record the number of faces, edges and vertices on the table in their notebooks.
5. Using the number of faces and previous knowledge of polyhedron nomenclature, students should name the figure.
6. After the data are collected, each group should begin a discussion to determine the mathematical relationship between the three variables observed: faces, edges and vertices.
7. This discussion should continue until the students arrive at a pattern that governs each example observed. **Teacher Note:** Even if the pattern is quickly found by some, allow all groups adequate time to arrive at their own conclusions.

8. Once the pattern is observed by most groups, write Euler's formula on the board: *Faces + Vertices = Edges + 2*.
9. After the activity, gather the class together and pose the following question: "What does Euler's formula define as true for all convex polygons?"
10. Finally, move to the "Activity Discussion and Processing" section of the activity.

Assessment

- Ask students to find various convex polyhedra in the classroom and demonstrate that Euler's formula applies
- Have students carefully graph a polyhedron that was not built in the lesson. Ask them to apply Euler's formula

Activity Discussion and Processing

To close the lesson, end with a group discussion about what was learned during the activity. Circle up the group, and work through the following questions. If possible, record the group's responses on flip chart paper so all comments are displayed.

- What is the relationship established by Euler's formula between the number of faces, edges and vertices found in a convex polyhedron?
- Why does Euler's formula not apply to concave polyhedra? Use an example to illustrate
- How can the relationship established in Euler's formula be used in everyday situations?
- How does Euler's formula enhance mathematical communication?

Here are some additional topics for discussion:

- The definition of a polyhedron
- The difference between concave and convex polyhedra
- Deducing and defining the mathematical relationship between the number of faces, edges and vertices found in Euler's formula
- The team effort used in analytical and deductive reasoning to communally discern Euler's formula

Activity Variations

1. **A different view.**

Challenge groups to build a concave polyhedron with Toobeez. After recording the required data, demonstrate that Euler's formula does not apply. **Teacher Note:** If time does not permit building a model, have students carefully draw a concave polyhedron for homework on graph paper and conduct the same analysis.

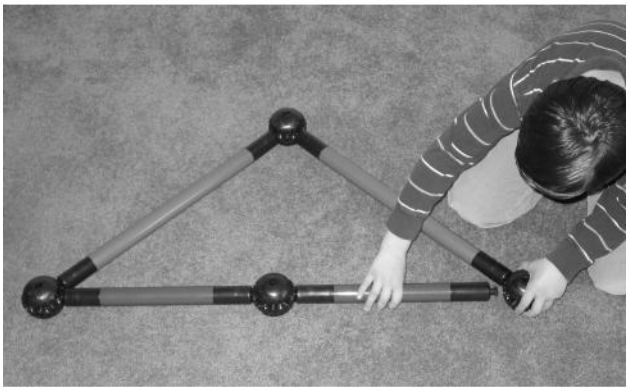
2. **Extension/Follow up.**

Have the students refer to their textbooks or a provided formula list and calculate the volume for various polyhedron models.

Activity Notes

ACTIVITY 9

ISOSCELES INTRODUCTION



Objectives

- Introduce and define the isosceles triangle
- Define and model the basic method to determine the area of isosceles triangles
- Introduce and practice more complex approaches in determining the area of isosceles triangles
- Work with others in a cooperative learning style

Preparation

Setup Time: 5 minutes
Mini-Lesson Time: 25 minutes
Materials:

1 Toobeez set per group
1 meter stick
1 notebook per student
pen or pencil

Activity Plan

Time: 50 - 60 minutes
Space: Medium
Instruction: Whole class, groups and individuals

Character Focus

Teamwork/Cooperation

The Challenge

Students will be able to calculate the area of isosceles triangles without directly measuring the height.

Activity Setup

1. Teachers should construct a model of an isosceles triangle. Two sides of this triangle should be built using two 24" Toobeez tubes. The third side should consist of two 16" Toobeez tubes with a sphere in the center connecting to the other two tubes in a triangular arrangement.

Activity Mini-Lesson

1. The class should be led in a discussion to determine the unique qualities of the model. **Teacher Note:** Students should realize that two sides of the triangle are equal in length.
2. After the class arrives at the appropriate observations, teachers should present the definition of an isosceles triangle on the board. Students should copy this definition in their notebooks.
3. Ask the class to recall any math strategies used in studying triangles. **Teacher Note:** The desired answer is the Pythagorean theorem. If this has not previously been studied, precede this lesson with Activity #2 "Pythagorean Puzzler," which offers an excellent introduction.
4. After the class recalls the Pythagorean theorem, ask the class whether that theorem can be directly applied to the model of the isosceles triangle. **Teacher Note:** Students should realize the model is not a right triangle and the Pythagorean theorem does not apply.
5. Teachers should introduce the area equation for triangles:
   $$Area = \frac{1}{2} base \times height$$
6. Ask the class the strategy for determining the base length.
7. After students realize the base can be directly measured, have a volunteer measure the model's base length with a meter stick and report the result to the class.
8. Ask the class: "Can any side of this model be measured to determine the height of the triangle?" Allow a discussion so ideas can be introduced.
9. Once it is agreed upon that no side can be directly measured to determine height, the teacher should insert a 16" Toobeez tube extending from the center sphere of the base to the sphere at the apex of the triangle.